IB MYP YEAR 5

ASSESSMENT TASK

# A Broad-based Test

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| **Subject:** | Y10 ***Extended*** Mathematics | **Name : (Class)** | Sharon Lau. (10T) |
| **Assessment:** | **Broad-based Test** | | |
| **Topics covered** | **Polynomials, Transformations (and transforming functions), Probability, Vectors, Matrices, Indices** | | |
| **Date of assessment:** | **Thursday 16th February 2012** | | |

- **This task assesses Criteria A and C;**
- **Time allowed – *one hour 40 minutes;***
- **You must answer <u>all</u> the questions;**
- **Write your answers in the spaces provided;**
- **Show all of your working – <u>not</u> just the answer**
- **GDCs are allowed.**

**Criterion A**

| Levels | Task-Specific Rubric | Official IB Descriptors |
|---|---|---|
| 0 | The student does not reach a standard described by any of the descriptors given below. | |
| 1-2 | Students are reasonably successful with the Part A questions only. Any errors here are relatively minor. | The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts. |
| 3-4 | Students are successful with Part A questions. The only errors in part B questions are minor. | The student generally makes appropriate deductions when solving **more complex** problems in **familiar** contexts. |
| 5-6 | Students are successful with Part A and B questions. The only errors in part C questions are minor. | The student **generally** makes appropriate deductions when solving **challenging** problems in a **variety** of **familiar** contexts. |
| 7-8 | Students are successful with Part A, B and C questions. The only errors in part D questions are minor. | The student **consistently** makes appropriate deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar situations**. |

**Criterion C**

| Levels | Task-Specific Rubric | Official IB Descriptors |
|---|---|---|
| 0 | The student does not reach a standard described by any of the descriptors given below. | |
| 1-2 | Very little working is shown, and/or the steps shown are confusing. Only the most basic mathematical symbols are used with accuracy. | The student shows **basic use** of mathematical language and/or forms of mathematical representation. The lines of reasoning are **difficult to follow.** |
| 3-4 | The working shown is generally adequate. Only a few errors in symbols/terminology are evident. It is reasonably easy to follow a student's logic/reasoning. | The student shows **sufficient use** of mathematical language and forms of mathematical representation. The lines of reasoning are **clear though not always logical or complete.** The student moves between different forms of representation with **some success.** |
| 5-6 | There are very few, if any, errors in symbols/terminology. All steps in calculations are shown in their completeness. It is easy to follow all the student's logic/reasoning. | The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise, logical** and **complete.** The student moves **effectively** between different forms of representation. |

## Part A (Level 1-2 Questions)

**Q1.** Factorise ***completely*** the following expressions:

**(a)** $x^2 - 16$

$x^2 - 4^2$

$(x+4)(x-4)$ //

Answer (a) $(x+4)(x-4)$ //

**(b)** $x^2 - 16x$

$x^2 - 2^4 x$

$x(x - 2^4)$

Answer (b) $x(x-2^4)$ //

**(c)** $x^2 - 16x - 36$

$x \quad -18$

$x \quad +2$

$(x-18)(x+2)$

Answer (c) $(x-18)(x+2)$

**Q2.** Simplify the following expressions, giving your answers in the form $a^n$, where a and n are integers:

**(a)** $3^5 \times 3^{-4}$

Answer (a) $3^1$

**(b)** $2^4 \times 3^4$

$(2^2)^2 \times (3^2)^2$.

Answer (b) $(2^2)^2 \times (3^2)^2$.

**(c)** $(6^2)^3$

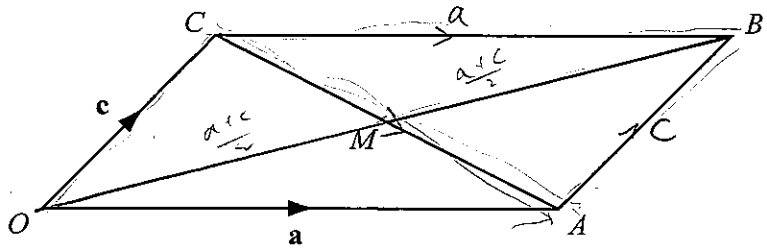
Answer (c) $6^6$

**(d)** $16^2 \div 2^8$

$\frac{(2^4)^2}{2^8} = \frac{2^8}{2^8}$

Answer (d) $2^0$

**Q3.**




$\overrightarrow{OB} = a + c$

OABC is a parallelogram. $\overrightarrow{OA}$ = **a**, $\overrightarrow{OC}$ = **c** and M is the midpoint of CA. Find, in terms of **a** and **c**:

**(a)** $\overrightarrow{OB}$

$\overrightarrow{OA} + \overrightarrow{AB}$

Answer (a) ..... $a + c$ .....

**(b)** $\overrightarrow{CA}$

$\overrightarrow{CB} + \overrightarrow{BA}$

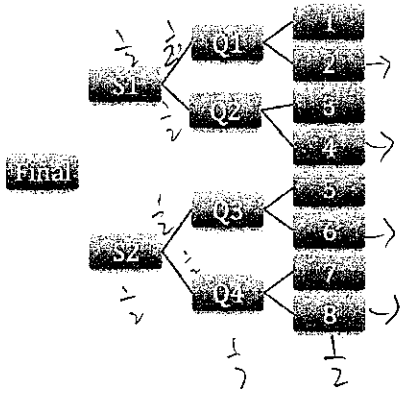
Answer (b) ..... $-(a + c)$ .....

**(c)** $\overrightarrow{BM}$

Answer (c) ..... $-\left(\frac{a+c}{2}\right)$ .....

## Part B (Level 3-4 Questions)

**Q4**. Eight teams take part in a basketball tournament. Each team is equally likely to win any particular game. Winning teams advance to the next stage. The losers go home!!




$\frac{4}{8}$

**(a)** Find the probability that all the even numbered team reach the quarter-finals (Q1, Q2, Q3, Q4).

$P\left(\frac{4}{8}\right)$

$\Rightarrow P\left(\frac{1}{2}\right)$,

Answer (a) $P\left(\frac{1}{2}\right)$

**(b)** Find the probability that team 1 will play team 8 in the final.

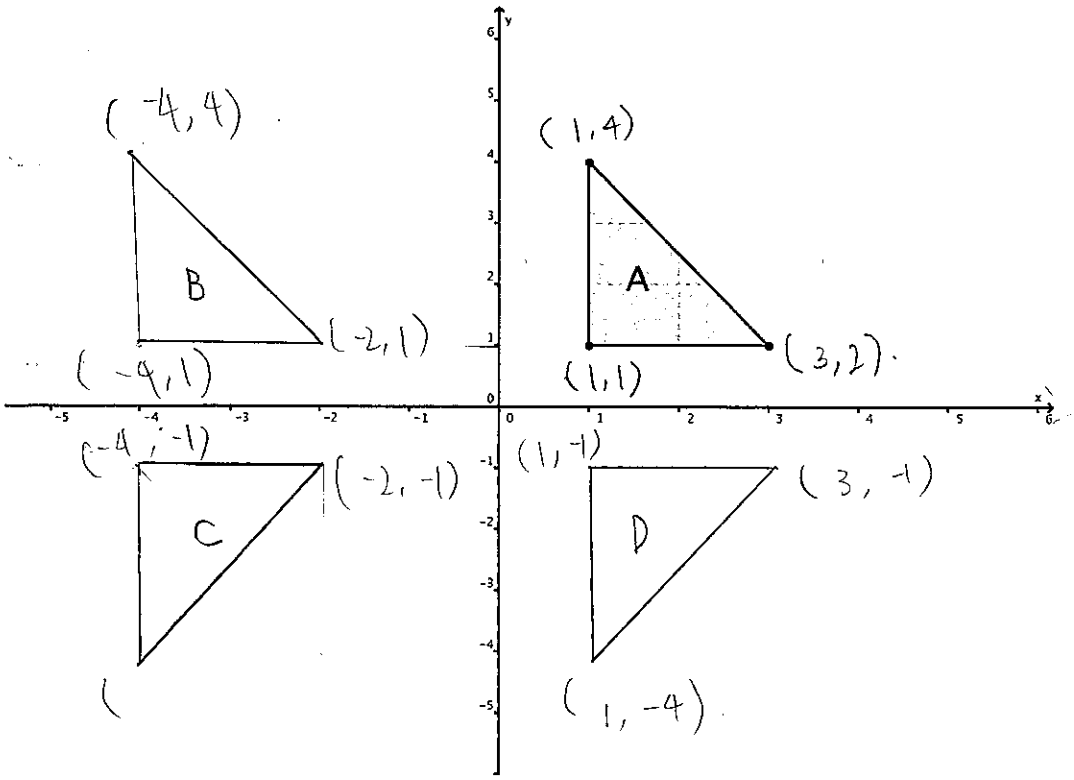
$\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$

$= \frac{1}{8} + \frac{1}{8}$

$= \frac{2}{8} = \frac{1}{4}$

Answer (b) $P\left(\frac{1}{4}\right)$

**Q5.** Triangle A is plotted on the Cartesian plane below.




**(a)** On the same Cartesian plane above, draw the following:

**(i)** Triangle A after it is translated by a translation vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$, label it B.

**(ii)** Triangle C is the reflection of triangle B in the *x*-axis, draw triangle C.

**(iii)** Triangle D is the reflection of triangle A in the *x*-axis, draw triangle D.

**(b)** Describe the single transformation, which maps triangle C onto triangle D.

shift trangle C to the right in 3 units.

Answer f(x-3)

**Q6.** Erik runs a race at an average speed of x m/s.
His time is $(3x - 9)$ seconds and the race distance is $(2x^2 - 8)$ metres.

**(a)** Write down an equation in x and show that it simplifies to
$x^2 - 9x + 8 = 0$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(8)}}{2(1)}$$

**(b)** Solve $x^2 - 9x + 8 = 0$

$$\Rightarrow x = \frac{9 \pm \sqrt{81 - 32}}{2} \qquad \Rightarrow x = 8 \text{ , } x = 1 \text{ ,,}$$

$$\Rightarrow x = \frac{9 \pm \sqrt{49}}{2}$$

$$\Rightarrow x = \frac{9 \pm 7}{2}$$

Answer (b) ..... $x = 8$ , $x = 1$ .....

**(c)** Write down Erik's time and the race distance.

Time : $(3 \times 8) - 9$

: 15 seconds.

Race distance : $(2(8)^2) - 8$.

$= 128 - 8$

$= 120$ m

Answer (c) ..... 15 seconds, 120 m. .....

## Part C (Level 5-6 Questions)

**Q7.** Luis deposits a large sum of money in a bank account that pays 0.6% interest, compounded monthly. How long does it take Luis's money to grow by 10%?

Answer .....

**Q8.** A is the matrix $\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix}$ and AB is the matrix $\begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$. Find the matrix B.
(Remember the order of matrix multiplication matters!)

let matrix B be $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$$\begin{pmatrix} 5 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5a+2b & 5c+2d \\ 2a+0 & 2c+0 \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 44 & 8 \end{pmatrix}$$

$\Rightarrow 5a+2b = 11$ - ①, $5c+2d = 2$ - ③

$\Rightarrow 2a+0 = 44$ - ②, $5c+0 = 8$ - ④

$\Rightarrow 2a = 44$, $a = 22$ - ⑤ $\Rightarrow 5c = 8$, $c = 1.6$ - ⑥

sub ⑥ in ③.

$\Rightarrow 5(1.6)+2d = 2$.

$8+2d = 2$.

$2d = -6$.

$d = -3$.

sub ⑤ in ①.

$5(22)+2(b) = 11$

$110+2b = 11$

$2b = -99$

$b = -49.5$

Answer $B = \begin{pmatrix} 22 & 1.6 \\ -49.5 & -3 \end{pmatrix}$

**Q9.** Solve the following equation for x:

$$4^{x+1} + 4^x + 4^{x-1} = 42$$

$4^x \times 4^1 + 4^x + 4^x \times 4^{-1} = 42$.

$4^x\left(4+1+\frac{1}{4}\right) = 42$.

~~$4^x\left(5\frac{1}{4}\right) = 42$~~

$4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

Answer $x = \frac{3}{2}$

## Part D (Level 7-8 Questions)

**Q10.** You are given the quadratic equation $x^2 + kx + 4 = 0$

**(a)** What values of k ensure that this equation has repeated real roots?

$$x^2 + kx + 4 = 0$$
$$kx + 4 = -x^2$$
$$kx = -x^2 - 4$$
$$k = \frac{-(x^2+4)}{x}$$

Answer (a) $k = \frac{-(x^2+4)}{x}$

**(b)** It is decided that k is an integer. A student chooses a value of k at random from the interval

$-5 \leq k \leq 5$

What is the probability that the resulting quadratic equation has no real roots?

(-5), (-4), (-3), (-2), (-1), (0), -1, 2, 3, 4, 5

$P\left(\frac{6}{11}\right)$

Answer (b) $P\left(\frac{6}{11}\right)$

**(c)** You are told to transform the original quadratic to $y = x^2$ **in one step.** What value of k would you choose, and what transformation would you perform?

I would perform enlargement

Answer (c) enlargement,

**Q11.** $A = \begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} x \\ y \end{pmatrix}$

We are told that AB = kB (where k is an integer).

**(a)** Using AB = kB, set up and simplify two simultaneous equations with x and y in.

$$AB = \begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} 1x + 6y \\ 4x + 3y \end{pmatrix}$$

$$k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1x + 6y \\ 4x + 3y \end{pmatrix}$$

$$\begin{pmatrix} kx \\ ky \end{pmatrix} = \begin{pmatrix} 1x + 6y \\ 4x + 3y \end{pmatrix}$$

$$kx = x + 6y \quad , \quad ky = 4x + 3y$$

**(b)** By solving the simultaneous equations in (a), find possible values for k.

$$k = \frac{x + 6y}{x} \quad , \quad k = \frac{4x + 3y}{y}$$

Answer .....

**NOW GO BACK AND CHECK YOUR WORK**